Conjunctive Queries for a Tractable Fragment of OWL 1.1

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ISWC 2007 + ASWC 2007

Presentation for the *ISWC 2007 paper of the same title (link)*

Motivation

Current developments in the OWL-world:

- **Conjunctive queries**

  $hasBrother(x, y), Professor(y),$
  $hasCoauthor(x, y), hasCoauthor(y, Erdös)$

  $\leadsto$ "SPARQL for OWL"

Motivation

Current developments in the OWL-world:

- **Conjunctive queries**

  $$hasBrother(x, y), Professor(y),$$
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  $\leadsto$ "SPARQL for OWL"

- **Complex role inclusion axioms**

  $$hasParent \circ hasBrother \sqsubseteq hasUncle$$

  $\leadsto$ RBox in addition to TBox, ABox

Conjunctive Queries in OWL

Conjunctive querying for DL knowledge bases can be hard:

- Conjunctive queries for $\mathcal{SHIQ}$: 2-ExpTime-complete [Glimm et al. IJCAI-07, Lutz DL-07]
- Conjunctive queries for $\mathcal{SHOIQ}$ (OWL DL): open

$\leadsto$ focus on relevant sub-problem:
query answering with complex role inclusion axioms

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$\rightsquigarrow$ focus on relevant sub-problem:
query answering with complex role inclusion axioms

The description logic $\mathcal{EL}^{++}$

- TBox-operators: $\sqcap$, $\exists$, $\top$, $\bot$, $\{a\}$ (nominals)
- complex role inclusion axioms (RBox)

$\rightsquigarrow$ polynomial time reasoning possible

A simple $\mathcal{EL}^{++}$ example

$C \sqsubseteq \exists R.D$

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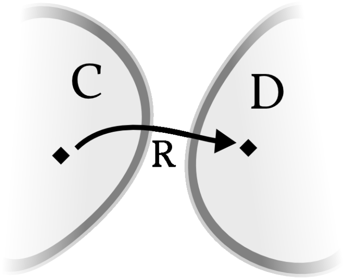

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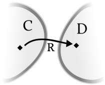
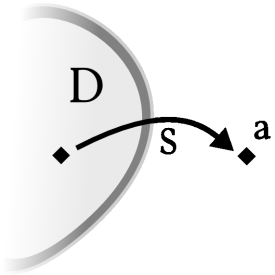
$C \sqsubseteq \exists R.D \qquad D \sqsubseteq \exists S.\{a\}$





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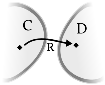

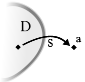

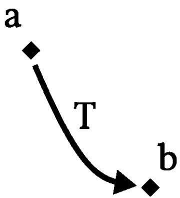

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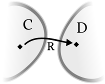

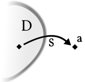

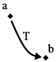

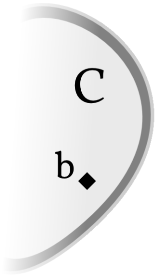

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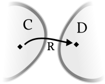
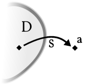
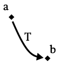
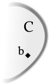
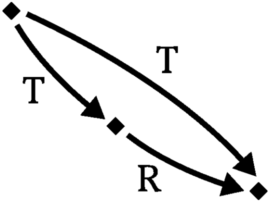




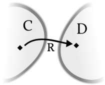




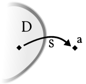


Reasoning with automata

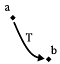
$C \sqsubseteq \exists R.D$



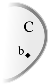
$D \sqsubseteq \exists S.\{a\}$



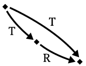
$\{a\} \sqsubseteq \exists T.\{b\}$



$\{b\} \sqsubseteq C$



$T \circ R \sqsubseteq T$



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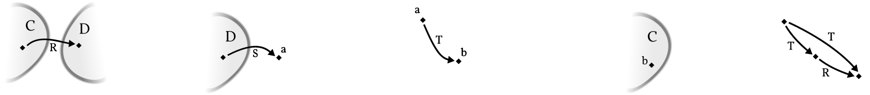

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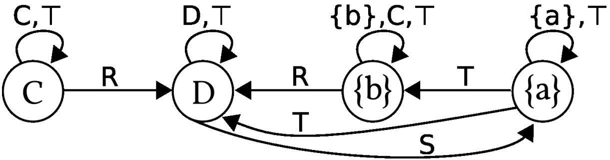

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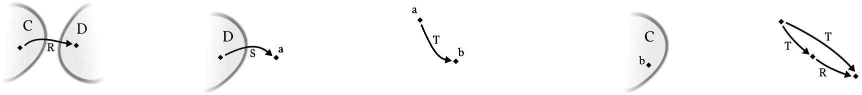

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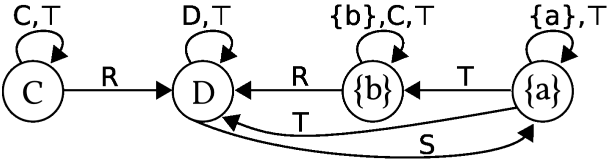

Example: start state $\{b\}$, end state D

- "*RD*"
- "*CRD*"
- "*RSTR*"

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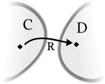

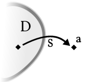

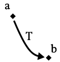

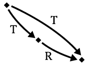

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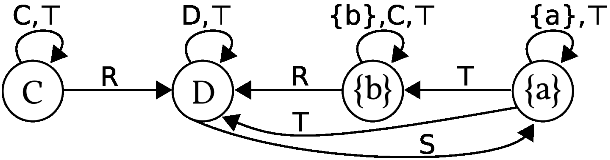

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- "RD" $\quad\mapsto\quad$ $\{b\} \sqsubseteq \exists R.D$
- "CRD"
- "$RSTR$"

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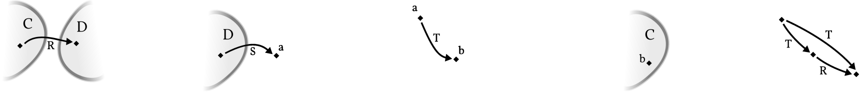

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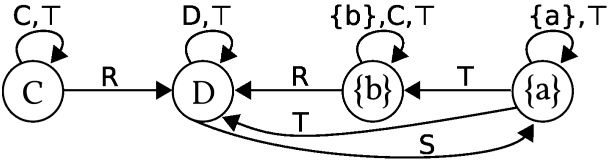

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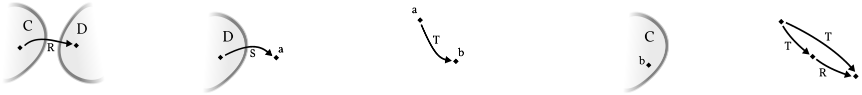

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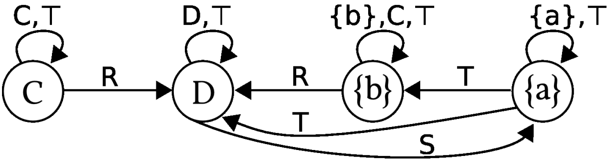

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Query answering – Part 1

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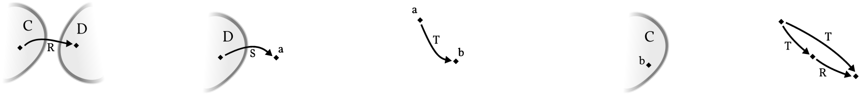

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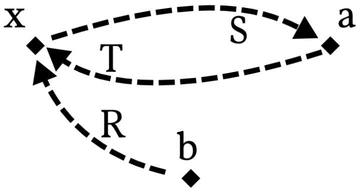

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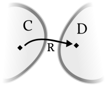

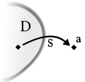

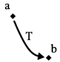

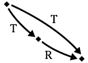

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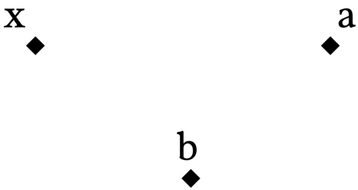

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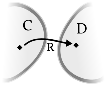

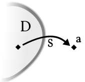

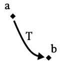

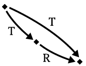

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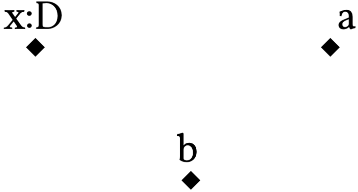

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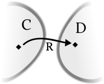

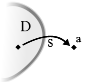

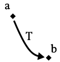

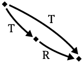

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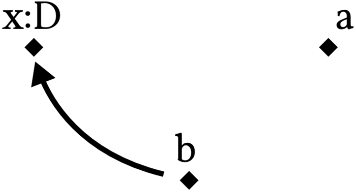

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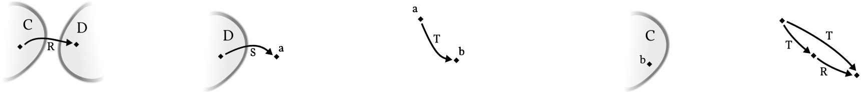

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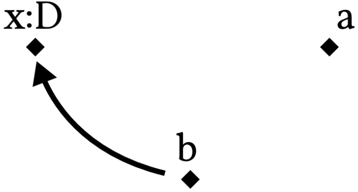

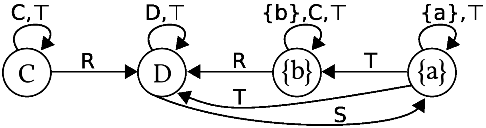

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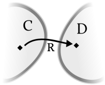

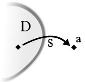

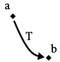

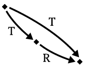

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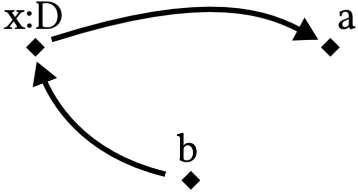
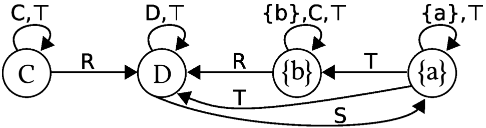
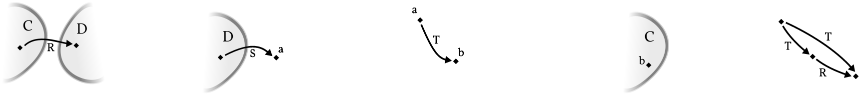
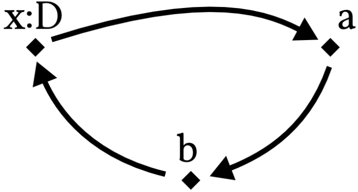


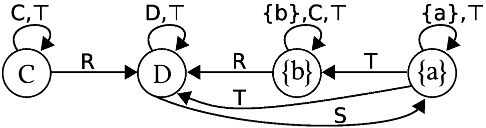


$C \sqsubseteq \exists R.D$ $\quad D \sqsubseteq \exists S.\{a\}$ $\quad \{a\} \sqsubseteq \exists T.\{b\}$ $\quad \{b\} \sqsubseteq C$ $\quad T \circ R \sqsubseteq T$



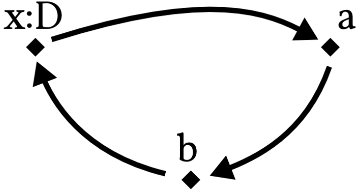
Example query: $S(x, a), T(a, x), R(b, x)$





Query answering – Part 2

We still need to check the role statements $S(x, a)$, $R(b, x)$, $T(a, x)$.



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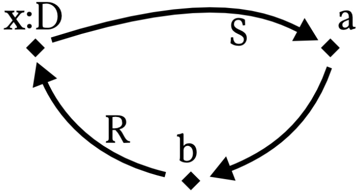

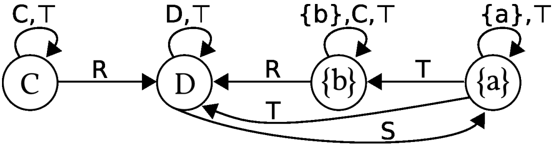

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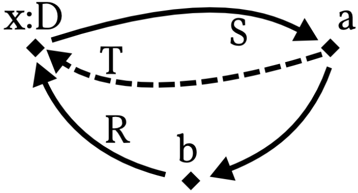

How can we show that $T(a, x)$?

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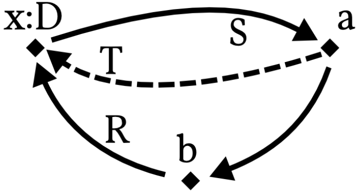

How can we show that $T(a, x)$?
Use RBox: $T \circ R \sqsubseteq T$

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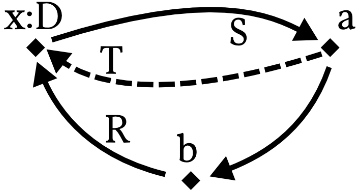
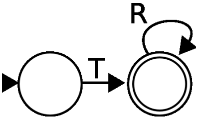


How can we show that $T(a, x)$?
Use RBox: $T \circ R \sqsubseteq T$ $\quad\rightsquigarrow$ use an automaton:



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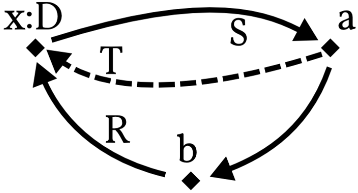
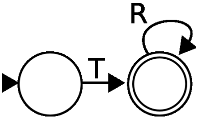
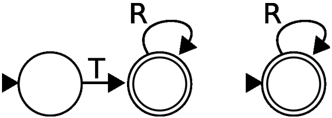


How can we show that $T(a, x)$?
Use RBox: $T \circ R \sqsubseteq T$ $\rightsquigarrow$ use an automaton:



…and split it:



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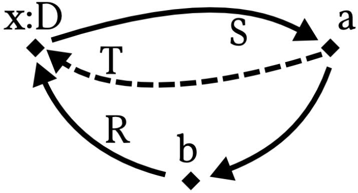
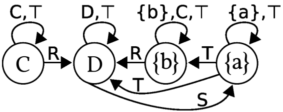
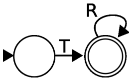


Check automata intersections:

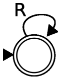


with



(path $a \to b$)

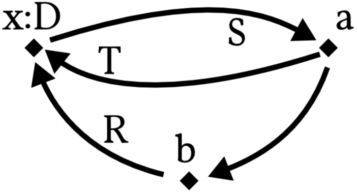
with



(path $b \to x$)

Query answering – Part 2

We still need to check the role statements $S(x, a)$, $R(b, x)$, $T(a, x)$.



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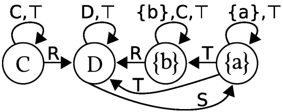
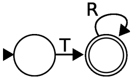
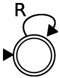


with



(path $a \rightarrow b$)

with



(path $b \rightarrow x$)

Summary

Algorithm summary:

1. Factorise query *(not in example)*
2. Guess node labels (classes) and generation paths
3. Check guesses via automaton
4. Guess complex role decompositions (split automaton)
5. Check guesses via automata intersection

Does this decide queries for $\mathcal{EL}^{++}$?

Summary

Algorithm summary:

1. Factorise query *(not in example)*
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Does this decide queries for $\mathcal{EL}^{++}$? – Alas, no:

Conjunctive querying in $\mathcal{EL}^{++}$ is undecidable!

Restriction of RBox is required: $\mathcal{EL}^{++} \cap \mathcal{SROIQ} \mapsto \mathcal{ELRO}$

$\leadsto$ decision procedure for queries in $\mathcal{ELRO}$

Results

	Variable parts: Query	R	T	A	Complexity
Combined complexity	●	●	●	●	PSPACE-complete
Query complexity	●				NP-complete
Schema complexity		●	●	●	PTIME-complete
Data complexity				●	PTIME-complete

"Conjunctive querying in presence of complex role inclusion axioms is decidable – but hard even for tractable logics."

What's next?

- $\mathcal{SROQ}$ and $\mathcal{SRIQ}$.
- Devise implementation strategies/goal-directed approaches.