All Elephants are Bigger than All Mice

Description Logic Rules

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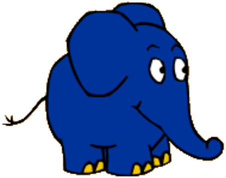

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Some simple statements:

- “Alkaline solutions neutralise acid solutions.”
- “Antihistamines alleviate allergies.”
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How can these be modelled in DL?
(In OWL DL and $\mathcal{SHOIQ}$ they cannot!)

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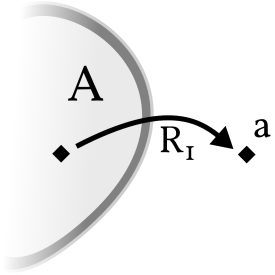

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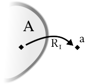

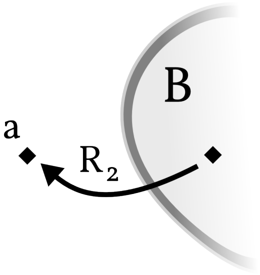

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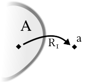

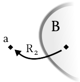

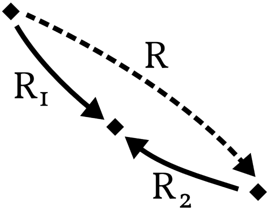

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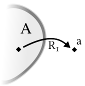
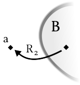


$B \sqsubseteq \exists R_2.\{a\}$



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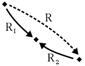

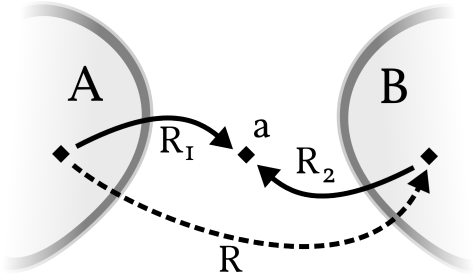

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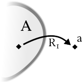

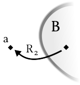

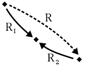

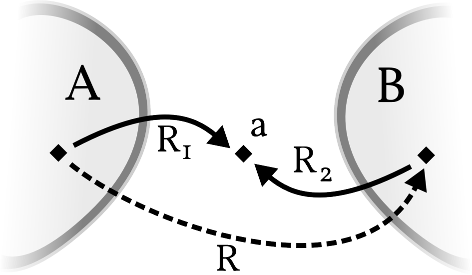

$\rightsquigarrow$ Polynomial transformation, no complexity issue

$\mathcal{SHOIQ}$, $\mathcal{SHOI}$, and $\mathcal{EL}^{++}$

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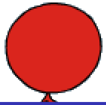

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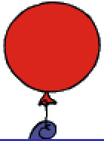

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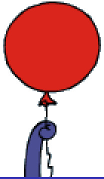

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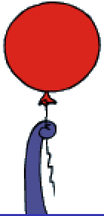

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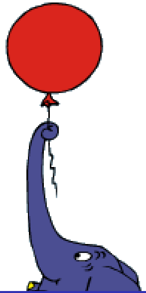

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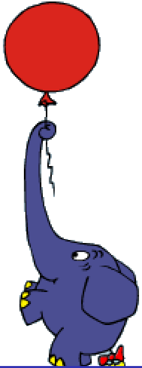

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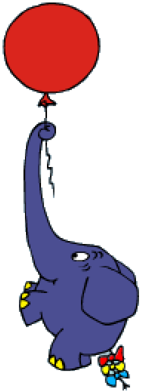

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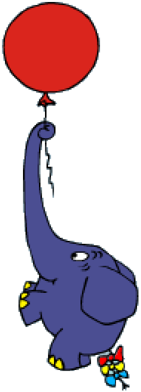

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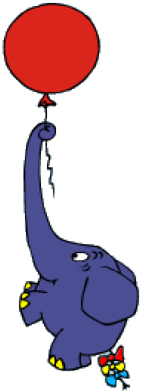

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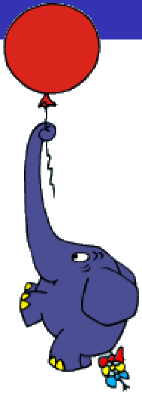

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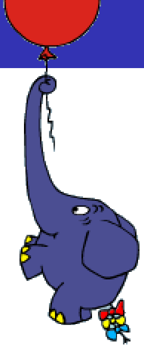

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Worst-case complexity for $\mathcal{SHOIQ}$, $\mathcal{SHOI}$, $\mathcal{EL}^{++}$ unaffected

A Rule Perspective

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Concept products can be expressed as rules:

$$A(x) \wedge B(y) \rightarrow R(x, y)$$

More examples of rules:

- $\text{Woman}(x) \wedge \text{hasChild}(x, y) \rightarrow \text{motherOf}(x, y)$
- $\text{Man}(x) \wedge \text{hasBrother}(x, y) \wedge \text{hasChild}(y, z) \rightarrow \text{Uncle}(x)$
- $\text{marriedTo}(x, y) \wedge \text{loves}(x, y) \rightarrow \text{Happy}(x)$

Modelling rules in $\mathcal{SROIQ}$

Some rules can be expressed in $\mathcal{SROIQ}$!

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  $\rightsquigarrow$ not possible in $\mathcal{SROIQ}$, but doable (for simple roles)

DL Rules

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Which rules can be expressed in $\mathcal{SROIQ}$?

Description Logic Rules

- SWRL rules (with complex concepts)
- rule bodies tree- or forest-shaped
- $R(x, x)$ and $R(x, y) \wedge S(x, y)$ in bodies only if R, S simple

$\mathcal{EL}$ Rules and DLP Rules

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DLP – Description Logic Programs

- Horn-logic fragment of OWL DL ($\mathcal{SHOIN}$)
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$\mathcal{EL}^{++}$ rules

- $\mathcal{EL}^{++}$ admits $\exists R.C$ in body and head
- has no native support for $\exists R.\mathit{Self}$, role conjunctions, U
- $\mathcal{EL}^{++}$ rules still polynomial

ELP: combination of $\mathcal{EL}^{++}$ rules and safe DLP rules – still polynomial

All Elephants are Bigger than All Mice

Conclusion

- Concept products: useful, easily explained modelling construct
- DL rules: non-boring FOL rule fragment of $\mathcal{SROIQ}$
- full reasoning support in any $\mathcal{SROIQ}$ reasoner
- no impact on worst-case complexity even in smaller DLs

Literature

- Rudolph, S.; Krötzsch, M.; and Hitzler, P. **All elephants are bigger than all mice.** In Proc. 21st Int. Workshop on Description Logics (DL-08). 2008. http://korrekt.org/page/Elephants.
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