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# ELP

## Tractable Rules for OWL 2

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# Where do we want to get to?



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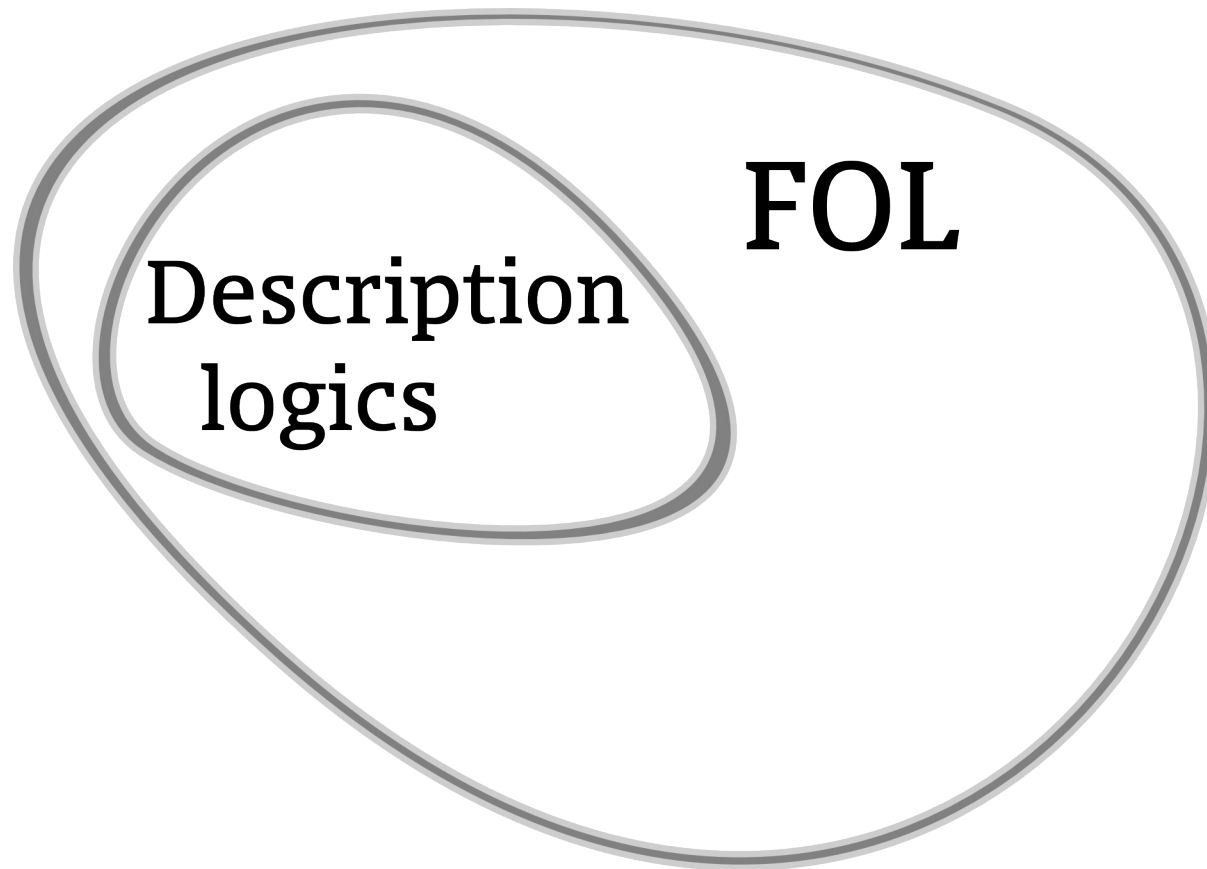


Semantic Web language that is ...

- focussed on data,
- rule-oriented,
- able to express schema knowledge,
- easy to implement,
- of polynomial worst-case complexity,
- compatible with OWL.

# OWL and Description Logics

# OWL and Description Logics



# OWL and Description Logics

“Sebastian ordered some Thai  
curry.”

*sebastian*:  $\exists \text{orderedDish}.\text{ThaiCurry}$

$\exists x. \text{orderedDish}(\text{sebastian}, x)$   
 $\wedge \text{ThaiCurry}(x)$

# OWL and Description Logics

“Everything ordered as a dish is actually a dish.”

$\top \sqsubseteq \forall \text{orderedDish.Dish}$

$\forall x. \forall y. \text{orderedDish}(x, y) \rightarrow \text{Dish}(y)$



# OWL and Description Logics

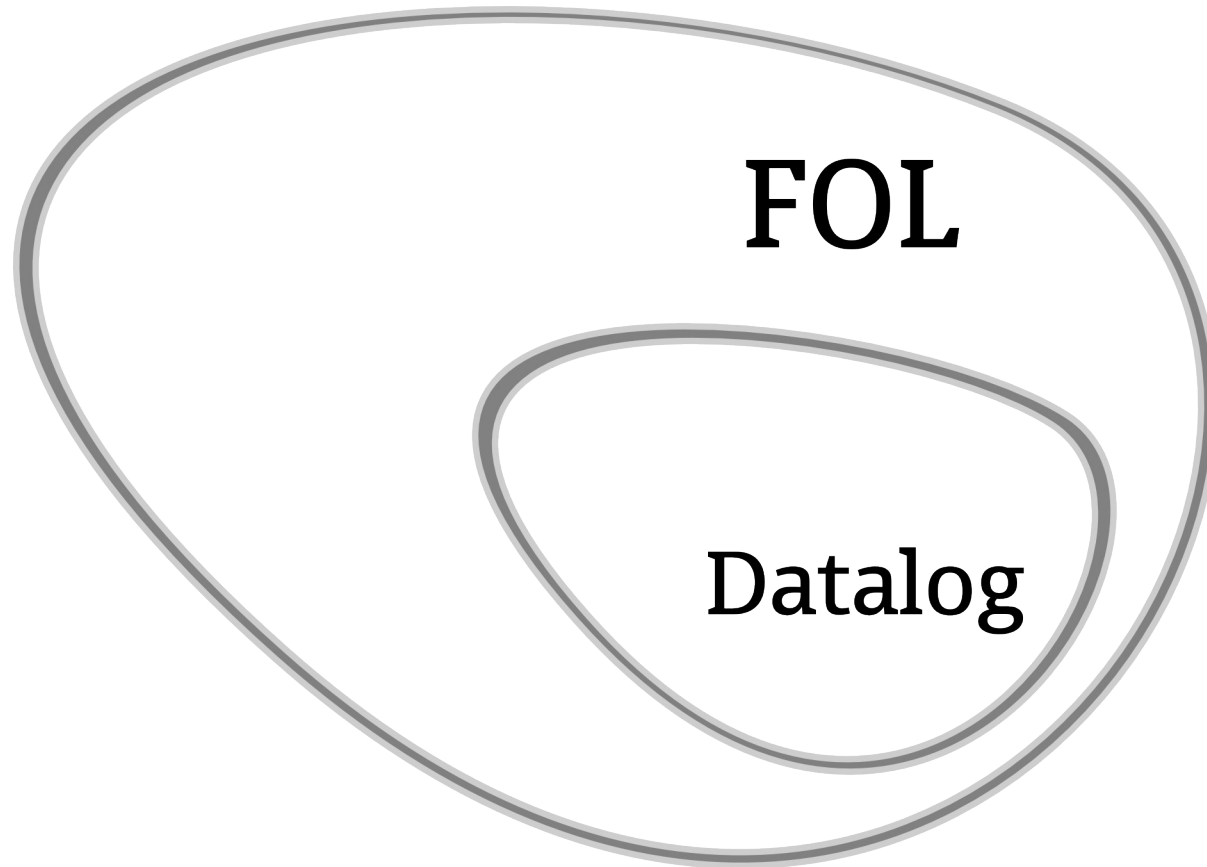
“Every Thai curry dish contains peanut oil.”

*ThaiCurry*  $\sqsubseteq$   $\exists$ *contains*.{*peanutOil*}

$\forall x. \textit{ThaiCurry}(x)$   
 $\rightarrow \textit{contains}(x, \textit{peanutOil})$

# Rules in First-Order Logic

# Rules in First-Order Logic



# Rules in First-Order Logic

“Nut allergics dislike nut products.”

$$\begin{aligned} \text{NutAllergic}(x) \wedge \text{NutProduct}(y) \\ \rightarrow \text{dislikes}(x,y) \end{aligned}$$

# Rules in First-Order Logic

“People who order a dish they dislike are unhappy.”

$$\text{orderedDish}(x,y) \wedge \text{dislikes}(x,y) \rightarrow \text{Unhappy}(x)$$

# Rules in First-Order Logic

“If someone dislikes an ingredient of a dish, she will also dislike the dish.”

$$\textit{dislikes}(x,z) \wedge \textit{Dish}(y) \wedge \textit{contains}(y,z) \rightarrow \textit{dislikes}(x,y)$$

# Rules in First-Order Logic

“Sebastian is a nut allergic, and peanut oil is a nut product.”

→ *NutAllergic(sebastian)*

→ *NutProduct(peanutOil)*

**Can we combine  
datalog rules and DL axioms?**



“

*ThaiCurry*  $\sqsubseteq \exists \text{contains}.\{\text{peanutOil}\}$

$\top$   $\sqsubseteq \forall \text{orderedDish.Dish}$

*sebastian*:  $\exists \text{orderedDish.ThaiCurry}$

$\text{NutAllergic}(x) \wedge \text{NutProduct}(y) \rightarrow \text{dislikes}(x,y)$

$\text{orderedDish}(x,y) \wedge \text{dislikes}(x,y) \rightarrow \text{Unhappy}(x)$

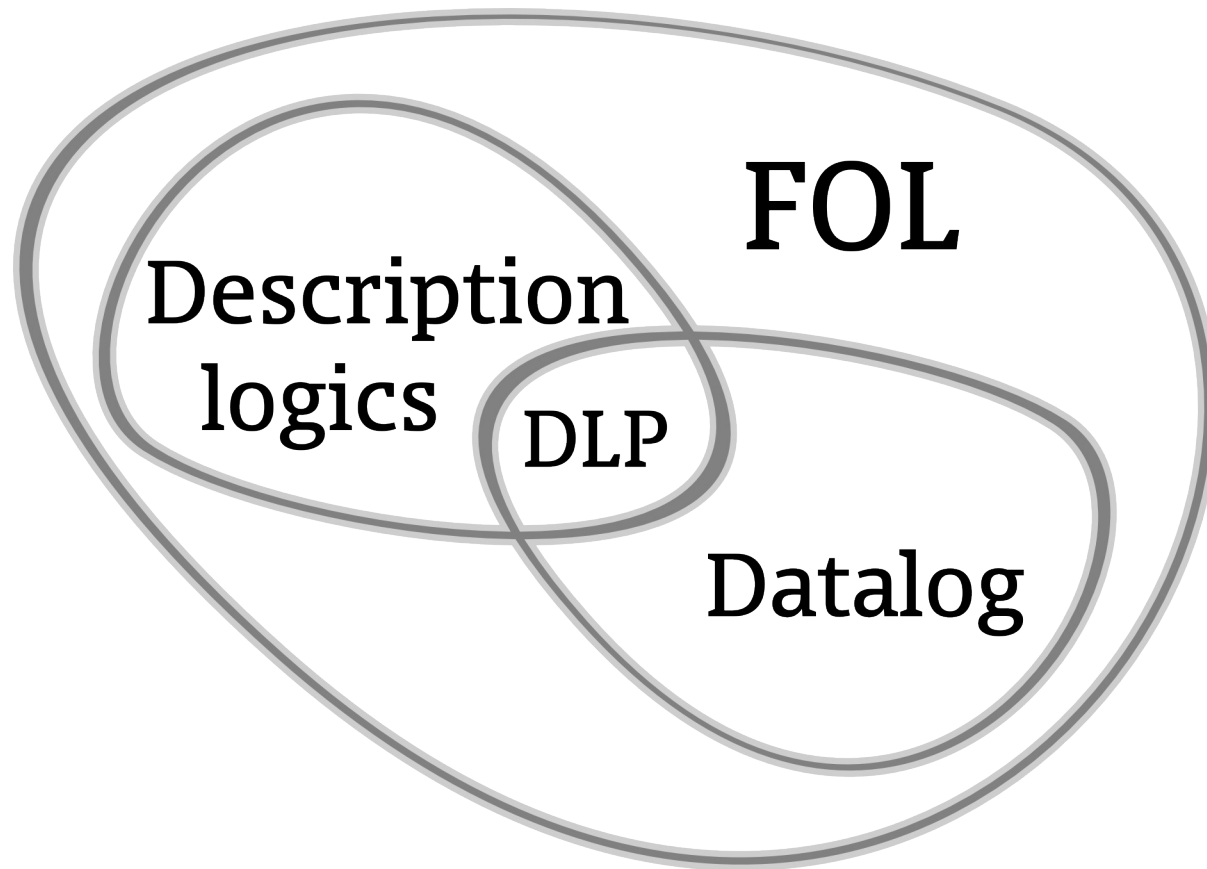
$\text{dislikes}(x,z) \wedge \text{Dish}(y) \wedge \text{contains}(y,z) \rightarrow \text{dislikes}(x,y)$

$\rightarrow \text{NutAllergic}(\text{sebastian})$

$\rightarrow \text{NutProduct}(\text{peanutOil})$

”

# Combining OWL and Rules



# DLP: “OWL $\cap$ datalog”

*ThaiCurry*  $\sqsubseteq \exists \text{contains}.\{\text{peanutOil}\}$  ✓

$\top$   $\sqsubseteq \forall \text{orderedDish.Dish}$  ✓

*sebastian*:  $\exists \text{orderedDish.ThaiCurry}$  ✗

*NutAllergic*(*x*)  $\wedge$  *NutProduct*(*y*)  $\rightarrow$  *dislikes*(*x*,*y*) ✗

*orderedDish*(*x*,*y*)  $\wedge$  *dislikes*(*x*,*y*)  $\rightarrow$  *Unhappy*(*x*) ✗

*dislikes*(*x*,*z*)  $\wedge$  *Dish*(*y*)  $\wedge$  *contains*(*y*,*z*)  $\rightarrow$  *dislikes*(*x*,*y*) ✗

$\rightarrow$  *NutAllergic*(*sebastian*) ✓

$\rightarrow$  *NutProduct*(*peanutOil*) ✓

# SWRL: “OWL $\cup$ datalog”

*ThaiCurry*  $\sqsubseteq \exists \text{contains}.\{\text{peanutOil}\}$  ✓

$\top$   $\sqsubseteq \forall \text{orderedDish}.\text{Dish}$  ✓

*sebastian*:  $\exists \text{orderedDish}.\text{ThaiCurry}$  ✓

*NutAllergic*(*x*)  $\wedge$  *NutProduct*(*y*)  $\rightarrow$  *dislikes*(*x*,*y*) ✓

*orderedDish*(*x*,*y*)  $\wedge$  *dislikes*(*x*,*y*)  $\rightarrow$  *Unhappy*(*x*) ✓

*dislikes*(*x*,*z*)  $\wedge$  *Dish*(*y*)  $\wedge$  *contains*(*y*,*z*)  $\rightarrow$  *dislikes*(*x*,*y*) ✓

$\rightarrow$  *NutAllergic*(*sebastian*) ✓

$\rightarrow$  *NutProduct*(*peanutOil*) ✓



**SWRL is undecidable.**

# DL-safe Rules

# DL-safe Rules

**Restrict rules to apply only to  
named individuals.**

# DL-safe Rules

*ThaiCurry*  $\sqsubseteq \exists \text{contains}.\{\text{peanutOil}\}$  ✓

$\top$   $\sqsubseteq \forall \text{orderedDish}.\text{Dish}$  ✓

*sebastian*:  $\exists \text{orderedDish}.\text{ThaiCurry}$  ✓

*NutAllergic*(*x*)  $\wedge$  *NutProduct*(*y*)  $\rightarrow$  *dislikes*(*x*,*y*) 🔒

*orderedDish*(*x*,*y*)  $\wedge$  *dislikes*(*x*,*y*)  $\rightarrow$  *Unhappy*(*x*) 🔒

*dislikes*(*x*,*z*)  $\wedge$  *Dish*(*y*)  $\wedge$  *contains*(*y*,*z*)  $\rightarrow$  *dislikes*(*x*,*y*) 🔒

$\rightarrow$  *NutAllergic*(*sebastian*) ✓

$\rightarrow$  *NutProduct*(*peanutOil*) ✓



# DL Rules

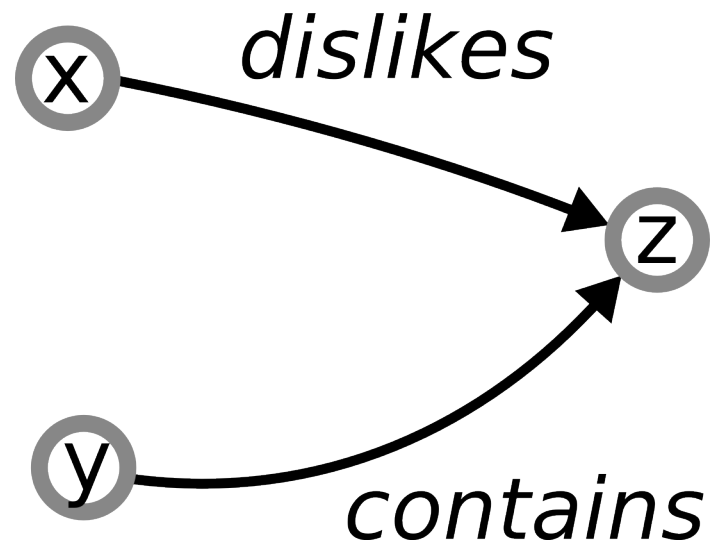
# DL Rules

Restrict to rules that could  
(indirectly) be encoded with DL  
anyway.\*

\*) rules with “tree-shaped” bodies

# DL Rules\*

$dislikes(x,z) \wedge Dish(y) \wedge contains(y,z) \rightarrow$   
 $dislikes(x,y)$



\* ) rules with “tree-shaped” bodies

# DL Rules

$ThaiCurry \sqsubseteq \exists contains.\{peanutOil\}$  ✓

$\top \sqsubseteq \forall orderedDish.Dish$  ✓

$sebastian: \exists orderedDish.ThaiCurry$  ✓

$NutAllergic(x) \wedge NutProduct(y) \rightarrow dislikes(x,y)$  ✓

$orderedDish(x,y) \wedge dislikes(x,y) \rightarrow Unhappy(x)$  ✗

$dislikes(x,z) \wedge Dish(y) \wedge contains(y,z) \rightarrow dislikes(x,y)$  ✓

$\rightarrow NutAllergic(sebastian)$  ✓

$\rightarrow NutProduct(peanutOil)$  ✓

# DL-safe rules + DL Rules

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$ThaiCurry \sqsubseteq \exists contains.\{peanutOil\}$  ✓

$\top \sqsubseteq \forall orderedDish.Dish$  ✓

$sebastian: \exists orderedDish.ThaiCurry$  ✓

$NutAllergic(x) \wedge NutProduct(y) \rightarrow dislikes(x,y)$  ✓

$orderedDish(x,y) \wedge dislikes(x,y) \rightarrow Unhappy(x)$  🚫

$dislikes(x,z) \wedge Dish(y) \wedge contains(y,z) \rightarrow dislikes(x,y)$  ✓

$\rightarrow NutAllergic(sebastian)$  ✓

$\rightarrow NutProduct(peanutOil)$  ✓



# DL-safe rules + DL Rules



Desired conclusion does not follow

It is still computationally expensive

DL-safe rules: ExpTime

DL Rules: like DL, i.e. NExpTime for OWL DL

# Tractable Profiles in OWL 2



# Tractable Profiles in OWL 2

**OWL RL:** *Horn logic* fragment,  
similar to DLP, no existentials

**OWL EL:** includes existentials, based  
on DL *EL++*

# Regaining Tractability: OWL 2 EL

$ThaiCurry \sqsubseteq \exists contains.\{peanutOil\}$  ✓

$\top \sqsubseteq \forall orderedDish.Dish$  ✓

$sebastian: \exists orderedDish.ThaiCurry$  ✓

$NutAllergic(x) \wedge NutProduct(y) \rightarrow dislikes(x,y)$  ✗

$orderedDish(x,y) \wedge dislikes(x,y) \rightarrow Unhappy(x)$  ✗

$dislikes(x,z) \wedge Dish(y) \wedge contains(y,z) \rightarrow dislikes(x,y)$  ✗

$\rightarrow NutAllergic(sebastian)$  ✓

$\rightarrow NutProduct(peanutOil)$  ✓

# Regaining Tractability: OWL 2 RL

*ThaiCurry*  $\sqsubseteq \exists \text{contains}.\{\text{peanutOil}\}$  ✓

$\top$   $\sqsubseteq \forall \text{orderedDish}.\text{Dish}$  ✓

*sebastian*:  $\exists \text{orderedDish}.\text{ThaiCurry}$  ✗

*NutAllergic*(*x*)  $\wedge$  *NutProduct*(*y*)  $\rightarrow$  *dislikes*(*x*,*y*) ✗

*orderedDish*(*x*,*y*)  $\wedge$  *dislikes*(*x*,*y*)  $\rightarrow$  *Unhappy*(*x*) ✗

*dislikes*(*x*,*z*)  $\wedge$  *Dish*(*y*)  $\wedge$  *contains*(*y*,*z*)  $\rightarrow$  *dislikes*(*x*,*y*) ✗

$\rightarrow$  *NutAllergic*(*sebastian*) ✓

$\rightarrow$  *NutProduct*(*peanutOil*) ✓

**OWL EL: PTime complete**



**OWL RL: PTime complete**

**OWL EL: PTime complete**



**OWL RL: PTime complete**



**OWL EL+RL:**

**N2ExpTime complete**

**Bringing it all together: ELP**

# Bringing it all together: ELP

DL Rules for OWL EL  
+  
Conjunctions of Roles\*  
+  
DL-safe *variables*

\* ) those roles must be “simple”



## Theorem

**Inferencing in ELP is PTime  
complete.**



# Bringing it all together: ELP

$ThaiCurry \sqsubseteq \exists contains.\{peanutOil\}$  ✓

$\top \sqsubseteq \forall orderedDish.Dish$  ✓

$sebastian: \exists orderedDish.ThaiCurry$  ✓

$NutAllergic(x) \wedge NutProduct(y) \rightarrow dislikes(x,y)$  ✓

$orderedDish(x,y) \wedge dislikes(x,y) \rightarrow Unhappy(x)$  ✓

$dislikes(x,z) \wedge Dish(y) \wedge contains(y,z) \rightarrow dislikes(x,y)$  ✓

$\rightarrow NutAllergic(sebastian)$  ✓

$\rightarrow NutProduct(peanutOil)$  ✓

# Bringing it all together: ELP

→ *Unhappy(sebastian)*



## Note

**ELP supports inferencing in  
OWL EL and OWL RL.**

# Understanding DL-safety

$\text{ThaiCurry} \sqsubseteq \exists \text{contains.} \mathbf{FishProduct}$

$\top \sqsubseteq \forall \text{orderedDish.Dish}$

**markus:**  $\exists \text{orderedDish.ThaiCurry}$

$\mathbf{Vegetarian}(x) \wedge \mathbf{FishProduct}(y) \rightarrow \text{dislikes}(x,y)$

$\text{orderedDish}(x,y) \wedge \text{dislikes}(x,y) \rightarrow \text{Unhappy}(x)$

$\text{dislikes}(x,z) \wedge \text{Dish}(y) \wedge \text{contains}(y,z) \rightarrow \text{dislikes}(x,y)$

$\rightarrow \mathbf{Vegetarian}(\text{markus})$

# Understanding DL-safety

*Unhappy(markus)*

cannot be concluded

# Towards Implementation



## Theorem

**Inferencing in ELP can be reduced in linear time to inferencing in 3-variable datalog.**

# Reasoning through Datalog

- Transformation to datalog is completely syntactic.
- Each axiom/rule can be transformed individually.
- Datalog engines can be used as blackbox.
- Instance and subsumption checking directly in datalog.





# Summary



ELP: DL-based tractable rule language

- Almost completely expressible in OWL 2
- Support for OWL EL and OWL RL
- Linear-time conversion to 3-var datalog  
→ simple implementation strategy

*Happy(markus)*

*Happy(sebastian)*

*Happy(pascal)*



[ Full paper available at <http://korrekt.org/page/ELP> ]